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BODY MOTION IN ARTIFICIAL THERMAL CHANNELS

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The possibility of reducing the losses of kinetic energy of small bodies that fly with high velocity in the atmosphere by creating a thermal channel along the flight trajectory owing to electromagnetic-energy supply from external sources is studied. The amount of energy necessary to change the ambient-medium parameters significantly is estimated, and an expression for the distribution of the temperature and density perturbations in the thermal wake of a system of crossed Gaussian laser beams, which form a region of heat release with an intensity one order of magnitude greater than the intensity of separate beams is derived. The spatial problem of the motion of a long solid body of revolution over a thermal channel is solved. The efficiency of the proposed method of decreasing the losses of kinetic energy along the trajectory is shown.

1. The problem of optimization of the parameters of body motion in the atmosphere with a view toward reaching a given region of space at an estimated moment of time with the minimum possible losses of kinetic energy has been of interest since the invention of Archimedes' launching devices. The traditional approach to its solution is to search for low-resistance bodies that are sufficiently stable and inertial and to update the means of launching. Attainment of the highest initial velocity is limited by various factors, including the strength of modern materials.

There is another method of improving of the motion parameters, namely, variation of the characteristics of the ambient medium along the flight path by outside energy supply to its sites along which the body flies. There are many studies (see, e.g., [1–7]) in which the influence of the energy release on the aerodynamic and momentum characteristics of streamlined bodies are investigated. Here the source of energy release can be both before the shock wave and behind it or the energy is additionally supplied along a definite part of the body surface. Although many interesting results were obtained, including the values of aerodynamic coefficients (see, e.g., [5]), the external energy action is not studied adequately, because this requires the solution of complex spatial problems of determination of the aerodynamic and moment characteristics of a body with allowance for energy supply and its motion in the disturbed atmosphere.

2. The present study deals with the effect of energy supply to the region located before the shock wave by means of devices positioned in immediate proximity from the initial point of flight trajectory of a body.

By analogy with the energy-supply method proposed in [6], we consider a system of N emitters of radius r_0, located over a ring of radius R (or inside this ring) in the plane of body start $X = 0$, where $X(t)$ is the current length of the path and $X_{\max} = X(t_{\max}) = L$ is the total path of the body in the atmosphere. Generally, dome-like radiation-intensity distributions according to, for example, the Gaussian law, are considered. The annular distributions with a fall-off in the intensity on the beam axis in supersonic and hypersonic streams at a certain distance Z along the beam path become dome-like [8, 9] in the neighborhood of the peak intensity.

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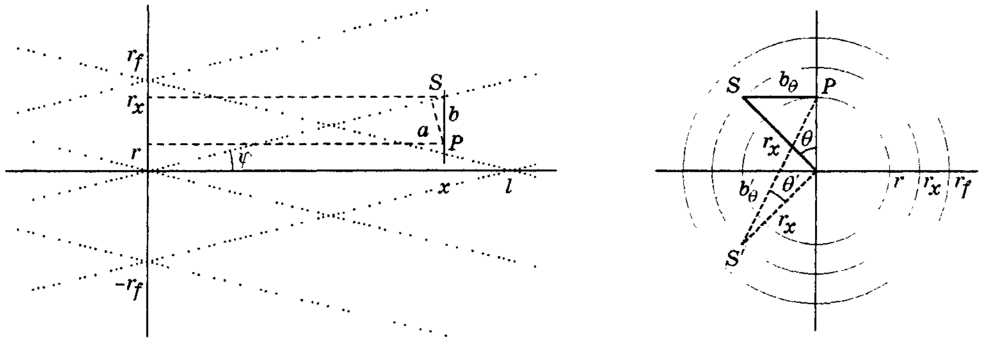

Fig. 1. Integration scheme of thermal sources.

This occurs in collimated and focused beams in the presence and absence of thermal self-action. In addition, in the case of a uniform initial distribution of the focused-beam intensity in the focal plane, we obtain a domed distribution in the main maxima in the form of a first-order Bessel function of the coordinate $J_1(r)$ transverse to the beam [10].

Let the radius of a separate beam in the focus be r_f, and the intensity distribution be Gaussian: $I(r)/I_i = \exp(-(r/r_f)^2)$, where $I_i = W\exp(-\alpha Z)/(N\pi r_f^2) = I_*/N$ is the characteristic intensity of a separate beam, W is the total initial power of the system of beams, $Z = \sqrt{X^2 + R^2}$ is the length of the beam path from the plane of start to the focus, $I_* = W\exp(-\alpha Z)/(\pi r_f^2) = NI_i$ is the characteristic intensity of the system of beams, and α is the coefficient of air-absorbed radiation. For a radiation wavelength equal to 10.6 μm (CO_2 laser), at sea level under normal atmospheric conditions the absorption coefficient equals $\alpha = 3 \cdot 10^{-4}$ m^{-1} and it decreases with increase in height and decrease in air density. In the initial and all subsequent cross sections, there are $n = N/(2\pi)$ beams per unit angular size of the circular arc along which the beams are located. The system of laser beams should be switched on in a certain time interval t after the start, such that $X(t) \gg R$, because the energy supply is not very efficient at smaller distances. The angle φ between the flight trajectory of the body and the path of the separate beam is small during the entire flight and is $\varphi \approx \sin\varphi \approx \tan\varphi \approx R/X \ll 1$. The heat-release region is shaped like a thin spindle strongly elongated along the flight trajectory and is upstream. In the plane X–Z, the heat-supply region looks like a rhombus with its diagonals equal to $2r_f$ and $2l$, where $l \approx r_f X/R \gg r_f$. We introduce a coordinate system x, r that moves together with the body with velocity $V(t)$, such that $x \sim l \ll X$ and $r \sim r_f \ll r_0 \ll R$, its initial point is at the center of the heat-release region, and the x axis is directed toward the body.

The problem of plane (two-dimensional) flow about a thermal source has already been considered for the case of continuous [11] and pulse-periodic [12] radiation regimes. The flow problem for a spindle-shaped thermal source was solved by Kuznetsov et al. in [6] for a continuous radiation regime and a homogeneous intensity distribution over the cross section of a separate beam. Here we consider beams whose intensity is distributed by the Gaussian law. The dimensionless heat-release intensity $q(r) = \alpha I(x,r)/(\alpha I_i N) = I(x,r)/I_*$ at the point $P(x,r)$, where $I(x,r)$ is the radiation intensity of the system of beams, is an integral of the contribution $nd\theta$ of separate beams located on the element of the arc $d\theta$ at the point $S(x, r_x)$ that is taken over the entire circle of radius $r_x \approx x\varphi \approx xr_f/l = xR/X$ (Fig. 1):

$$q(x,r) = \frac{I}{I_*} = \int_0^{2\pi} \exp\Big(-\Big(\frac{b_\theta}{r_f}\Big)^2\Big)\frac{1}{2\pi}\,d\theta = \exp\Big(-\Big(\frac{r}{r_f}\Big)^2 - \Big(\frac{x}{l}\Big)^2\Big) I_0\Big(\frac{2rx}{r_f l}\Big).$$

Here $I_0(2rx/(r_f l))$ is a modified zero-order Bessel function and $b_\theta = \sqrt{r_x^2 + r^2 - 2r_x r\cos\theta}$ is the distance between the points S and P. Allowance was made that $a \approx b_\theta$ for any θ. After the replacements $u = 2r_x r\cos\theta/r_f^2$ and $u = 2r_x r\sin\theta/r_f^2$, the integral becomes a tabular integral (see [13], p. 352]):

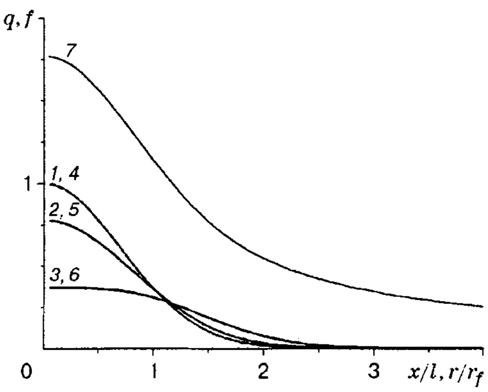

Fig. 2. Powers of the sources q in the heat-release region vs. the longitudinal coordinate x for $r/r_f = 0$, 0.5, and 1 (curves 1–3, respectively) and the transverse coordinate r (curves 4–6, respectively) and the relative normalized change in air density in the wake $f(r)$ (curve 7).

$$\int_0^a \frac{\cosh(bu)}{(a^2 - u^2)^{1/2}}\,du = \frac{\pi}{2}\,I_0(ab).$$

Figure 2 shows the function q vs. the longitudinal x (curves 1–3) and transverse r (curves 4–6) coordinates. In these coordinates, curves 1 and 4, 2 and 5, and 3 and 6 coincide pairwise.

Since the half-body equivalent to a thermal source is thin ($r_f/l \ll 1$), pressure perturbations are absent. The temperature perturbations are an integral of thermal sources along the streamlines (along the coordinate x), and the density perturbations are equal to the temperature perturbations with the opposite sign in the linear approximation of the perturbation theory. The expression for the perturbed density in the wake of a thermal source, which is an artificial channel of decreased density, can be written in the form

$$\frac{\rho}{\rho_\infty} = 1 - Qf\Big(\frac{r}{r_f}\Big), \qquad Q = \frac{\alpha I_* \exp(-\alpha Z) r_f}{\rho_\infty C_p T_\infty R}\,\frac{X(t)}{V(t)} = A\,\frac{X(t)}{V(t)t_*}\exp(-\alpha Z),$$
$$A = \frac{\alpha I_* r_f t_*}{\rho_\infty C_p T_\infty R}. \tag{1}$$

Here t_* is the characteristic time of motion (for example, $t_* = L/V_0$); the quantity $A/t_* = \alpha I_* r_f/(\rho_\infty C_p T_\infty R)$ (C_p is the specific heat at constant pressure, ρ_∞ and T_∞ are the density and temperature of the incoming flow, respectively, and R is a gas constant) characterizes the "reduced" heat-supply power relative to the enthalpy of undisturbed air.

The expression for the function $f(r/r_f)$ in formula (1) is written in the form

$$f\Big(\frac{r}{r_f}\Big) = \int_{-\infty}^{+\infty} q(x, r)\,d(x/l) = \sqrt{\pi}\exp\Big(-\frac{r^2}{2r_f^2}\Big)\,I_0\Big(-\frac{r^2}{2r_f^2}\Big).$$

In the calculations, we used the tabular integral (see [14, p. 306])

$$\int_0^\infty \exp(-px^2) I_\nu(cx)\,dx = \frac{1}{2}\sqrt{\frac{\pi}{p}}\exp\Big(\frac{c^2}{8p}\Big) I_{\nu/2}\Big(\frac{c^2}{8p}\Big).$$

We note that $I_0(0) = 1$; for large values of the argument, the modified Bessel function behaves asymptotically, $I_\nu(\zeta) \approx \exp(\zeta)/\sqrt{2\pi\zeta}\,(1 + O(1/\zeta))$ (see, e.g., [15]).

The function $f(r/r_f)$ is plotted in Fig. 2 (curve 7).

3. In solving the problem of body motion in the atmosphere, the following conditions are usually imposed: the small loss of kinetic energy on the flight trajectory and the passage through a definite part of it on a given time interval with a required degree of deviation from the initial direction of motion. The first condition is satisfied if the ballistic coefficient is $k = \rho_\infty C_x S/(2m) \ll 1$ (C_x is the drag coefficient and S and m are the maximum midsection and the mass of the body). To satisfy this condition, the smallness of the drag coefficient C_x is primarily required. If the nose of a body undergoes ablation, the drag coefficient of the body can increase by one to two orders of magnitude during the flight. The mass of the body decreases in this case, which also leads to an increase in k, though, this increase is not significant. Thus, the problem of ablation reduction is important. To satisfy the second condition, there should be sufficient stability in relation to the initial perturbations and the motion perturbations on the flight trajectory.

Thus, one can distinguish three stages in the solution of the problem of body motion: the choice of the material and shape of a body, determination of the aerodynamic and ablation characteristics, and calculation of the spatial motion in the presence of various disturbing factors. Studies have shown that the conditions listed above are fulfilled for pointed bodies of great elongation and with a stabilizing tail unit. Below, the examples of calculation refer to the case where the initial mass is $m_0 = 20$ g and a large part of the casing, including the nose, is made from a heavy material of density $\rho = 17$ g/cm^3. The tail-unit parameters were chosen with allowance for the requirement that the tail unit ensure a sufficient stabilizing pitching moment in any flight mode. In the calculations, we used the model of mass ablation employed in [6]. The aerodynamic characteristics of the body were found on the basis of the inviscid spatial flow calculation results from [16] with subsequent corrections that take into account the effect of viscosity within the framework of the boundary-layer theory with allowance for friction and the displacement thickness on the body and the tail unit.

The longitudinal and lateral body motions were calculated with the use of the spatial technique of [17] in the atmosphere with a density distribution simulated by expression (1) for various values of the dimensionless heat-release parameter A. Here we solved a system of complete equations of body motion with six degrees of freedom under the action of aerodynamic forces and gravity written in a coupled coordinate system for convenience of calculation of the aerodynamic loads:

$$\frac{d\boldsymbol{r}}{dt} = P\cdot\boldsymbol{V}, \quad \frac{d\boldsymbol{V}}{dt} = \frac{1}{m}\boldsymbol{F} = \frac{1}{m}(\boldsymbol{F}_a + P^{-1}\cdot\boldsymbol{F}_{gg}), \quad \frac{d\omega_\xi}{dt} = \frac{M_\xi}{I_\xi} - \frac{I_\zeta - I_\eta}{I_\xi}\omega_\zeta\omega_\eta, \quad \xi = x, y, z,$$
$$\frac{d\boldsymbol{i}_g}{dt} = -\boldsymbol{\omega}\times\boldsymbol{i}_g, \quad \frac{d\boldsymbol{j}_g}{dt} = -\boldsymbol{\omega}\times\boldsymbol{j}_g, \quad \boldsymbol{k}_g = \boldsymbol{i}_g\times\boldsymbol{j}_g. \tag{2}$$

Here $\boldsymbol{r}$ is the radius vector of the center of mass of a body in the terrestrial coordinate system, $\boldsymbol{V}$ is the velocity vector of the center of mass of the body, $\boldsymbol{F}$ is the principal vector of external forces, $\boldsymbol{F}_a$ is the vector of aerodynamic forces, $\boldsymbol{F}_{gg}$ is the gravity vector (in the terrestrial coordinate system), ω_x, ω_y, and ω_z are the components of the angular-velocity vector $\boldsymbol{\omega}$, M_x, M_y, and M_z are the components of the external (aerodynamic) moment, I_x, I_y, and I_z are the main central inertia moments of the body; (ξ, η, and ζ) is the cyclic permutation (x, y, and z), and $P = \|P_{ij}\|$, where $i, j = x, y, z$ is the transition matrix from coupled to terrestrial coordinates, whose elements P_{ij} are the unit-vector components of the axes $\boldsymbol{i}_g$, $\boldsymbol{j}_g$, and $\boldsymbol{k}_g$.

Equations (2) are supplemented by the equations of change of the state and shape of a body in flight, owing to ablation in particular. Considering that the condition of a body and the law of its change can be described by a finite number of parameters, we write these equations in the form

$$\frac{d\Phi}{dt} = \dot{\Phi}(\Phi, \rho, a, \boldsymbol{V}, \boldsymbol{\omega}). \tag{3}$$

Here Φ are the indicated parameters of state, ρ and a are the air density and the velocity of sound, respectively, in the neighborhood of the body, and $\dot{\Phi}(\Phi, \rho, a, \boldsymbol{V}, \boldsymbol{\omega})$ is a specified function of the arguments, whose form is determined by the adopted model of ablation. In particular, in the simplest case where the one-parameter semiempirical model of nose ablation is used and all changes in the aeroballistic characteristics of a body are caused by mass ablation, Eq. (3) can be written as follows:

$$\frac{d\Delta L}{dt} = f(r_c(\Delta L), \rho V^2/2, H),$$

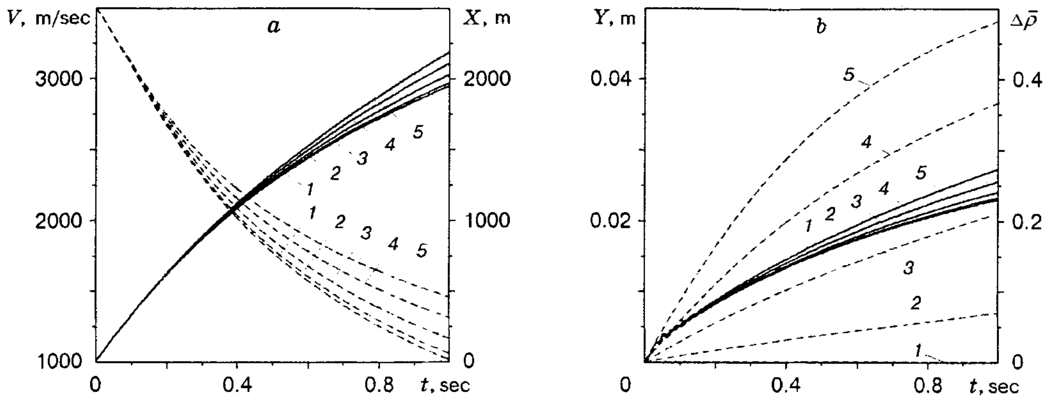

Fig. 3. Body motion in a thermal channel of radius 0.03 m (the initial angle of attack is $\alpha_0 = 3°$ and the characteristic flight time $t_* = 1$ sec): (a) the longitudinal coordinate X (solid curves) and the velocity of the body V (dashed curves) vs. the time t; (b) the lateral deviation of the body Y (solid curves) and the relative density changes $\Delta\bar{\rho}$ (dashed curves) vs. the time t: $A = 0$ (1) and $A/t_* = 0.0507$ (2), 0.1689 (3), 0.3378 (4), and 0.5067 sec (5).

where ΔL is a parameter that characterizes the degree of nose charring and, as this parameter, one can use the change in the total length of the body, $r_c(\Delta L)$ is the radius of bluntness of the nose, and $\rho V^2/2$ and H are the velocity head and complete enthalpy of the incoming stream.

The mass, the inertia moments, and the aerodynamic coefficients are calculated as functions of the body-state parameters:

$$m = m(\Phi), \quad I_x = I_x(\Phi), \quad I_y = I_y(\Phi), \quad I_z = I_z(\Phi),$$
$$F_{a\xi} = \frac{\rho_\infty V_\infty^2}{2} SC_\xi(\Phi, \rho_\infty, V, \omega), \quad M_\xi = \frac{\rho_\infty V_\infty^2}{2} SLm_\xi(\Phi, \rho_\infty, V, \omega), \quad \xi = x, y, z. \tag{4}$$

In this formulation, because of the influence of the thermal channel, the parameters of the flow around the body are coordinate functions. In the present study, this influence is connected mainly with the density variation described by expression (1).

The initial conditions for Eqs. (2)–(4) are set in the form

$$t = 0, \qquad x = x_0, \qquad y = y_0, \qquad z = z_0,$$
$$V_\xi = V_{\xi 0}, \quad \omega_\xi = \omega_{\xi 0}, \quad P_{\eta\xi} = P_{\eta\xi 0}, \quad \Phi = \Phi_0, \quad \xi = x, y, z, \quad \eta = x, y. \tag{5}$$

Problem (2)–(5) is solved numerically by the Runge–Kutta method.

Figure 3 shows calculation results obtained for the spatial motion of a body at various values of the power of emitters and a 0.03-m-wide heat-release region. The velocity and the passed path (Fig. 3a) vs. time, the lateral deviations vs. the initial direction of motion, and the relative density of a gas flowing around the body vs. time (Fig. 3b) are plotted. These data illustrate the effect of the reduced heat-supply power, which is characterized by the parameter A [see (1)], on the trajectory parameters. With increase in A (in particular, with increase in the heat-release power or with pressure drop in the ambient medium), the deceleration weakens. The mass ablation and, as a consequence, the gain in the drag coefficient decelerate considerably. Thus, the intense heat supply by means of electromagnetic radiation in this case allows one to increase substantially the effective flying range characterized by a finite velocity.

We also analyzed the effect of the lateral deviation of a body from the axis of the thermal channel on the trajectory parameters. As a result of the increase in the angle of attack, the body reaches the channel periphery, i.e., the lateral deviation Y increases. The degree of rarefaction of the medium in which the body travels decreases. Therefore, the velocity V and the flying range X decrease.

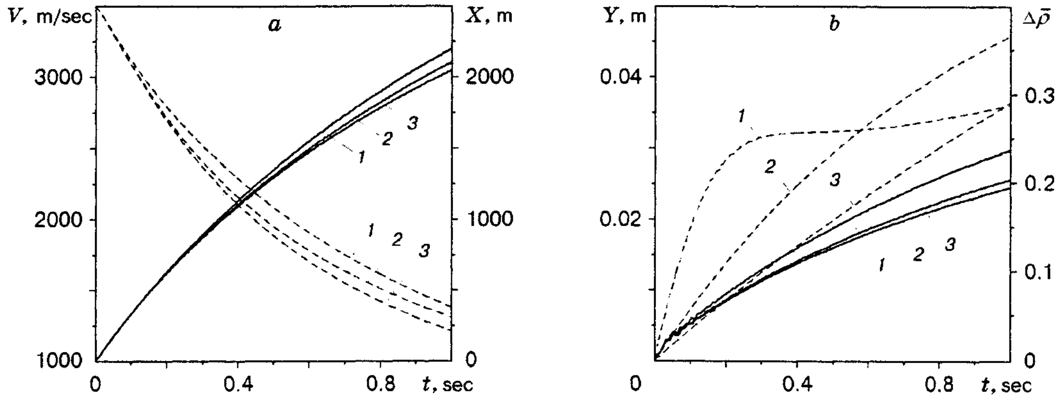

Fig. 4. Body motion in the thermal channel for $\alpha_0 = 3°$ and $t_* = 1$ sec: curves 1 refer to $r_f = 1$ cm and $A/t_* = 1.0133$ sec^{-1}, curves 2 to $r_f = 3$ cm and $A/t_* = 0.3378$ sec^{-1}, and curves 3 to $r_f = 5$ cm and $A/t_* = 0.2027$ sec^{-1} (for other notation, see Fig. 3).

Figure 4 shows the motion trajectories of a body for various values of the channel radius r_f. Here the total power of emitters and the undisturbed-atmosphere parameters (ρ_∞, T_∞, and α) and the emitter geometry remained constant. With decrease in the channel radius, the body reaches the lower-rarefaction region (and then the undisturbed atmosphere) and decelerates fast, as the lateral deviation increases. Obviously, the dimensions of the thermal channel should be correlated with the restrictions imposed on the lateral deviations of a body on the calculated site of the trajectory, because the unbounded increase in its width leads to an abrupt and unjustified growth in energy expenditures, and an extremely narrow channel is not effective. We also note that the unfavorable effect of the lateral deviations of the body can be easily eliminated by varying the law of deviation of emitters in accordance with trajectory measurement data obtained directly during the flight. In addition, the width of the heat-release region can be greatly enlarged with the use of longer-wavelength radiation, in particular, in the microwave range.

The calculation results show that one can decrease greatly the kinetic-energy losses of a flying body by means of thermal channels.

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